

# BARGAINING, MARKETS (& AUCTIONS)

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**Suppose you want to sell/buy  
one Microsoft stock, how would  
you do it?**

**Suppose you want to sell/buy an oil field, how would you do it?**

**Suppose you want to sell/buy a piece of art, what would be your preferred mechanism?**

# Introduction

- Auctions are widely studied economic mechanisms
  - Auctions refer to arbitrary resource allocation problems with self-motivated participants: Auctioneer and bidders
  - Auction (selling item(s)): one seller, multiple buyers  
*e.g., selling a CD on eBay*
  - Reverse auction (buying item(s)): one buyer, multiple sellers  
*e.g., procurement*
- ⇒ We will discuss auctions, but the same theory holds for reverse auctions

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## Historical note

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# Where auctions are used nowadays

- Treasury auctions (bill, notes, Treasury bonds, securities)
- Transfer of assets from public to private sector
  - Right to drill oil, off-shore oil lease
  - Use the 4G spectrum
- Government and private corporations (construction, education, etc.)
- Private firms sell products (flowers, fish, tobacco, livestock, diamonds, ...)
- Internet auctions
- Art auctions
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# Questions

- **Seller has information problem:** incomplete information about buyer's valuations (otherwise, he would just need to set price at maximum valuation of the buyer)
  - Which pricing scheme performs well in incomplete information settings?
  - Are auctions better suited for certain problems?
  - Does a specific type of auction yield greater revenue?
- **Buyer:** What are good bidding strategies?



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# How to compare auctions

## Revenue

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## Efficiency

The object ends up in the hands of the person, who values it the most (*resale does not increase efficiency*).

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# Open versus sealed bid auctions

## Open bid auction

Bidders (competitors) are informed of each other and do also observe each others behavior.

## Sealed bid auction (also closed bid auction)

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## Example: THE DUTCH AUCTION

Open descending auction where the auctioneer calls out a rather high price, lowering it until a player indicates his interest. The first player doing so wins the object to the given price.



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## Example: THE ENGLISH AUCTION

The counterpart to the Dutch auction. The auctioneer starts with a small price. By raising the price in small steps players indicate if they are still willing to pay the new price. It ends when only one person is in the game. He receives the object and pays the price at which the second last bidder dropped out at. (*e.g.: arts in an auction house*)



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## Further examples

### SEALED BID **FIRST** PRICE AUCTION

is a closed auction where the participant with the highest bid receives the good by paying his bid.

*(e.g., real estate auction via postal bidding)*

### SEALED BID **SECOND** PRICE AUCTION (VICKREY AUCTION)

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# Private versus common value

## Private value

The valuation of a bidder is independent of the valuations other bidders hold for the item. Further, no bidder knows with certainty the valuation of the other bidders.

## (Pure) common value

The (pure) common value is the same for every bidder, but bidders have different private information about what that value actually is.

*Example: In an auction of an oil field the amount of oil is unknown, but different bidders have different geological signals and learning another signal would change the valuation of a bidder.*

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# Private value auctions

## *Basic Auction Environment*

- Bidders  $i = 1, \dots, n$
- One object to be sold
- Bidder  $i$  observes a "signal"  $S_i \sim F(\cdot)$ , with typical realisation  $s_i \in [0, \bar{s}]$
- Bidder's signals  $S_1, \dots, S_n$  are independent
- Bidder  $i$ 's value  $v_i(s_i) = s_i$

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A set of auction rules will give rise to a game between bidders.

# Vickrey auction (sealed bid second price)

## *Auction Rules:*

- Bidders are asked to submit sealed bids  $b_1, \dots, b_n$
- Bidder who submits highest bid wins the object
- Winner pays the amount of the second highest bid

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## **Proof**

Bid  $b_i$  means  $i$  will win  $\iff$  the price is below  $b_i$

Bid  $b_i > s_i \Rightarrow$  sometimes  $i$  will win at price above value

Bid  $b_i < s_i \Rightarrow$  sometimes  $i$  will loose at price below value

# Vickrey auction: Expected revenue

- Seller's revenue equals second highest value.
- Let  $S^{i:n}$  denote the  $i$ th highest of  $n$  draws from distribution  $F$ .
- Seller's expected revenue is  $\mathbb{E} [S^{2:n}]$ .

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## Optimal bid for first price auction

Suppose bidders  $j \neq i$  bid  $b_j = b(s_j)$ ,  $b(\cdot)$  increasing.

Bidder  $i$ 's expected payoff:

$$U(b_i, s_i) = (s_i - b_i) \cdot Pr [b_j = b(S_j) \leq b_i, \forall j \neq i]$$

Bidder  $i$  chooses  $b_i$  to solve:

$$\max_{b_i} (s_i - b_i) F^{n-1}(b^{-1}(b_i))$$

where  $F(\cdot)$  is the probability that a random draw from  $F$  is smaller than  $\cdot$ .

First order condition (differentiate w.r.t.  $b_i$ ):

$$0 = (s_i - b_i)(n-1)F^{n-2}(b^{-1}(b_i))f(b^{-1}(b_i))\frac{1}{b'(b^{-1}(b_i))} - F^{n-1}(b^{-1}(b_i))$$

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Bidder  $i$  chooses  $b_i$  to solve:

$$\max_{b_i} (s_i - b_i) F^{n-1}(b^{-1}(b_i))$$

where  $F(\cdot)$  is the probability that a random draw from  $F$  is smaller than  $\cdot$ .

First order condition (differentiate w.r.t.  $b_i$ ):

$$0 = (s_i - b_i)(n-1)F^{n-2}(b^{-1}(b_i))f(b^{-1}(b_i))\frac{1}{b'(b^{-1}(b_i))} - F^{n-1}(b^{-1}(b_i))$$

and  $f = F'$ .

## Optimal bid for first price auction

$$0 = (s_i - b_i)(n - 1)F^{n-2}(b^{-1}(b_i)) f(b^{-1}(b_i)) \frac{1}{b'(b^{-1}(b_i))} - F^{n-1}(b^{-1}(b_i))$$

At symmetric equilibrium,  $b_i = b(s_i)$ , first order condition is (dropping subscript  $i$ ):

$$b'(s) = (s - b(s))(n - 1) \frac{f(s)}{F(s)}$$

This differential equation can be solved using the boundary condition  $b(0) = 0$ :

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## Expected First Price Auction Revenue

- Revenue is highest bid  $b(s^{1:n})$ ; expected revenue is  $\mathbb{E}[b(S^{1:n})]$ .

$$b(s) = s - \frac{\int_0^s F^{n-1}(\tilde{s}) d\tilde{s}}{F^{n-1}(s)} = \frac{1}{F^{n-1}(s)} \int_0^s \tilde{s} F^{n-1}(\tilde{s}) d\tilde{s} = \mathbb{E}[S^{1:n-1} | S^{1:n-1} \leq s]$$

That is, if a bidder has signal  $s$ , he sets his bid equal to the expectation of the highest of the other  $n - 1$  values, conditional on all those values being less than his own.

The expected revenue is:

$$\mathbb{E}[b(S^{1:n})] = \mathbb{E}[S^{1:n-1} | S^{1:n-1} \leq S^{1:n}] = \mathbb{E}[S^{2:n}]$$

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# Revenue equivalence theorem

## Theorem (Myerson 1981)

Suppose  $n$  bidders have private values  $s_1, \dots, s_n$  identically and independently distributed with cdf  $F(\cdot)$ .

Then any equilibrium of any auction game in which

- ① the bidder with the highest value wins the object,
- ② a bidder with value 0 gets zero profits,

generates the same revenue in expectation.

# Risk neutrality is necessary for revenue equivalence

- Risk-averse agents

- for bidders:

Dutch, first-price sealed-bid  $\geq$  Vickrey, English

Compared to a risk neutral bidder, a risk averse bidder will bid higher ("buy" insurance against the possibility of loosing)

*(Utility of winning with a lower bid < utility consequence of loosing the object)*

- For auctioneer:

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- The expected revenue in third-price is greater than the expected revenue in second-price (English)
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# Private value is necessary for revenue equivalence

## Results for non-private value auctions

- Dutch strategically equivalent to first price sealed bid
- Vickrey not strategically equivalent to English
- All four protocols (Dutch, English, Vickrey, first) allocate item efficiently

### Theorem: Revenue non-equivalence

With more than 2 bidders, the expected revenues are not the same:  
 $\text{English} \geq \text{Vickrey} \geq \text{Dutch} = \text{First-price sealed bid}$

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## A little auction

- We are playing a second price, sealed-bid auction
- You are bidding for a bag of coins with less than 10,000 Rappen
- The winner(s) will play the second highest bid and receive the amount in the jar
- Website: <https://scienceexperiment.online/classroom/r/eDhNWt>



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- Website: <https://scienceexperiment.online/classroom/r/SVB894>
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# The winner's curse

- In a common value auction each bidder must recognize that he/she wins the object only when he/she has the highest signal
- Failure to take into account the bad news about others' signals that come with any victory can lead, on average, to the winner paying more, than the prize is worth
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*multiple indistinguishable items for sale*

Examples:

- IBM stock
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- Auction of multiple, distinguishable items
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**Task:** The UK wanted (in 2000) to allocate “air space” for 3G mobile usage

### Why an auction is a good choice:

- Utility to companies unknown to government; auction is method most likely allocating resource to those who can use them most valuably (rather than for example a “competition”)
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Hal Varian leads the economics team designing the auctioning of ad-space.

- Multi-unit
- Multi-item
- Dynamic element (repeated games)
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>50 billion USD revenue per year

Hal Varian on Google auctions:

<https://www.youtube.com/watch?v=Pj0HTFRaBWA>

The screenshot shows a Google search for "adwords management service". The search results are displayed in a grid format. The top result is "AdWords Management Expert - Results" with a sub-heading "Guaranteed Within 30 Days" and a link to "Free Proposal & Analyst Call Now". Below this is another result for "PPC Management - \$234 / M - Reduce Costs, Increase Sales & ROI" from "AdWords/Yahoo Certified PPC Company". The search results are filtered by location to "Santa Monica, CA". On the right side, there are several sponsored ads, including "AdWords Management" from "Experienced Ad Campaign Management", "Managing Google AdWords" from "Enroll in The Free AdWords Training Program", "PPC Campaign Management" from "Lower Cost Per Lead. Improve CVRs.", "AdWords Management" from "Adwords Certified Partner", "AdWords Management" from "Spand Smirity to Dominate Search", and "Don't Sign Up For AdWords" from "Before You Check Out Jumpfly".

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- AdWords Management Expert - Results**: Guaranteed Within 30 Days. Free Proposal & Analyst Call Now. [silverbackshategies.com](#)
- PPC Management - \$234 / M - Reduce Costs, Increase Sales & ROI**: AdWords/Yahoo Certified PPC Company. [ebrandtz.com](#) (38 reviews)
- Certified AdWords® Pro - Most Trusted Agency**: Since 2005, 1 (877) 886 1070. Free AdWords & adCenter Evaluations. [google.mysadwordsexpert.com](#)
- Google Adwords Campaign Management Services - Adwords Professional**: Adwords Agency offers Google Adwords campaign management services. Paul Ransom is a professional Adwords account manager and consultant specializing in ... [www.adwordsagency.com](#) - Cached - Similar
- Adwords Management | PPC Management | Adwords Help | Adwords Tips**: Unlike Other PPC Management Services Out There, I'm Not Going to Charge an ... the only choice when it comes to outsourcing your Adwords and PPC management! ... [www.managemypaperclick.com](#) - Cached - Similar
- Adwords Management**: Experienced Ad Campaign Management Ad Spend > \$2k/month? You need us! [ppcmanagemnt.com/600-324-6921](#)
- Managing Google AdWords**: Enroll in The Free AdWords Training Program For SEM Professionals! [www.google.com](#)
- PPC Campaign Management**: Lower Cost Per Lead. Improve CVRs. Adwords Certified Partner. California. [bizwatchsearchanalytics.com](#)
- Adwords Management**: Adwords Management Service From American Express! Free for 15 days. [searchmanager.com](#)
- Adwords Management**: Spend Smarter to Dominate Search With The Adwords Management Experts. [www.saop.com](#)
- Don't Sign Up For AdWords**: Before You Check Out Jumpfly. Voted #1 AdWords Management Agency. [www.jumpfly.com/Google-AdWords](#)

## Some introductory texts

- Vijay Krishna: Auction Theory (Academic Press)
- Paul Klemperer: Auction Theory: A guide to the literature (Journal of Economics Survey)
- Tuomas Sandholm COURSE: CS 15-892 Foundations of Electronic Marketplaces (CMU)