BARGAINING, MARKETS (& AUCTIONS)

Heinrich H Nax & Heiko Rauhut

heinrich.nax@uzh.ch

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Suppose you want to sell/buy one Microsoft stock, how would you do it?

Suppose you want to sell/buy an oil field, how would you do it?

Suppose you want to sell/buy a piece of art, what would be your prefered mechanism?

Introduction

- Auctions are widely studied economic mechanisms
- Auctions refer to arbitrary resource allocation problems with self-motivated participants: Auctioneer and bidders
- Auction (selling item(s)): one seller, multiple buyers

  e.g., selling a CD on eBay
- Reverse auction (buying item(s)): one buyer, multiple sellers

  e.g., procurement

  $\Rightarrow$ We will discuss auctions, but the same theory holds for reverse auctions

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- 193 A.D. after having killed Emperor Pertinax, Praetorian guards sold the Roman Empire by means of an auction.

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Where auctions are used nowadays

- Treasury auctions (bill, notes, Treasury bonds, securities)
- Transfer of assets from public to private sector
  - Right to drill oil, off-shore oil lease
  - Use the 4G spectrum
- Government and private corporations (construction, education, etc.)
- Private firms sell products (flowers, fish, tobacco, livestock, diamonds, . . .)
- Internet auctions
- Art auctions
- Procurement

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Questions

- Seller has information problem: incomplete information about buyer's valuations (otherwise, he would just need to set price at maximum valuation of the buyer)
  - Which pricing scheme performs well in incomplete information settings?
  - Are auctions better suited for certain problems?
  - Does a specific type of auction yield greater revenue?

- Buyer: What are good bidding strategies?

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Revenue

The revenue for the seller is the expected selling price.

Efficiency

The object ends up in the hands of the person, who values it the most (*resale does not increase efficiency*).

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Open versus sealed bid auctions

Open bid auction

Bidders (competitors) are informed of each other and do also observe each others behavior.

Sealed bid auction (also closed bid auction)

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Example: The Dutch auction

Open descending auction where the auctioneer calls out a rather high price, lowering it until a player indicates his interest. The first player doing so wins the object to the given price.

Dutch flower auctions

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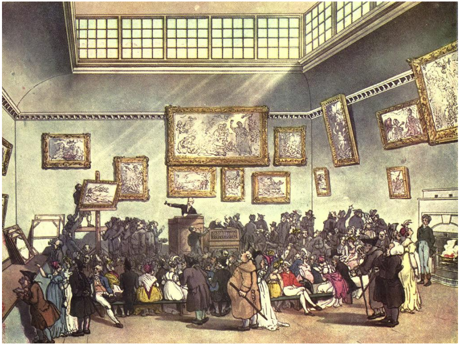
Example: THE ENGLISH AUCTION

The counterpart to the Dutch auction.The auctioneer starts with a small price. By raising the price in small steps players indicate if they are still willing to pay the new price. It ends when only one person is in the game. He receives the object and pays the price at which the second last bidder dropped out at. *(e.g.: arts in an auction house)*



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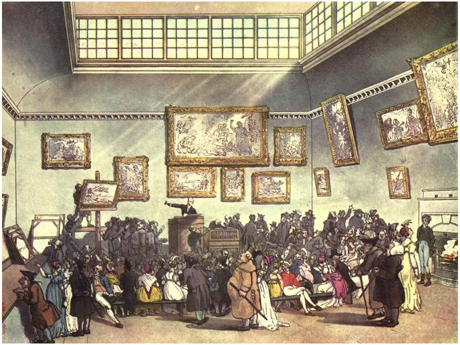

Further examples

SEALED BID **FIRST** PRICE AUCTION
is a closed auction where the participant with the highest bid receives the good by paying his bid.
(e.g., real estate auction via postal bidding)

SEALED BID **SECOND** PRICE AUCTION (VICKREY AUCTION)
is a closed auction where the participant with the highest bid receives the good by paying the second highest bid.
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Private versus common value

Private value

The valuation of a bidder is independent of the valuations other bidders hold for the item. Further, no bidder knows with certainty the valuation of the other bidders.

(Pure) common value

The (pure) common value is the same for every bidder, but bidders have different private information about what that value actually is.

Example: In an auction of an oil field the amount of oil is unknown, but different bidders have different geological signals and learning another signal would change the valuation of a bidder.

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Private value auctions

Basic Auction Environment

- Bidders $i = 1, \ldots, n$
- One object to be sold
- Bidder i observes a "signal" $S_i \sim F(\cdot)$, with typical realisation $s_i \in [0, \overline{s}]$
- Bidder's signals $S_i, \ldots, S_n$ are independent
- Bidder i's value $v_i(s_i) = s_i$

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A set of auction rules will give rise to a game between bidders.

Vickrey auction (sealed bid second price)

Auction Rules:

- Bidders are asked to submit sealed bids $b_1, \cdots, b_n$
- Bidder who submits highest bid wins the object
- Winner pays the amount of the second highest bid

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Bid b_i means i will win $\iff$ the price is below b_i

Bid $b_i > s_i \Rightarrow$ sometimes i will win at price above value

Bid $b_i < s_i \Rightarrow$ sometimes i will loose at price below value

Vickrey auction: Expected revenue

- Seller's revenue equals second highest value.
- Let $S^{i:n}$ denote the ith highest of n draws from distribution F.
- Seller's expected revenue is $\mathbb{E}\left[S^{2:n}\right]$.

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Bidders will want to shade bids below their values

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Optimal bid for first price auction

Suppose bidders $j \neq i$ bid $b_j = b(s_j)$, $b(\cdot)$ increasing.
Bidder i's expected payoff:

$$U(b_i, s_i) = (s_i - b_i) \cdot Pr\left[b_j = b(S_j) \leq b_i, \forall j \neq i\right]$$

Bidder i chooses b_i to solve:

$$\max_{b_i} (s_i - b_i) F^{n-1}\left(b^{-1}(b_i)\right)$$

where $F(\cdot)$ is the probability that a random draw from F is smaller than $\cdot$.
First order condition (differentiate w.r.t. b_i):

$$0 = (s_i - b_i)(n-1)F^{n-2}\left(b^{-1}(b_i)\right) f\left(b^{-1}(b_i)\right) \frac{1}{b'(b^{-1}(b_i))} - F^{n-1}\left(b^{-1}(b_i)\right)$$

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At symmetric equilibrium, $b_i = b(s_i)$, first order condition is (dropping subscript i):

$$b'(s) = (s - b(s))(n-1)\frac{f(s)}{F(s)}$$

This differential equation can be solved using the boundary condition $b(0) = 0$:

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Optimal bid for first price auction

$$0 = (s_i - b_i)(n-1)F^{n-2}\left(b^{-1}(b_i)\right) f\left(b^{-1}(b_i)\right) \frac{1}{b'(b^{-1}(b_i))} - F^{n-1}\left(b^{-1}(b_i)\right)$$

At symmetric equilibrium, $b_i = b(s_i)$, first order condition is (dropping subscript i):

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- We have found necessary conditions for symmetric equilibrium.
- Verifying that $b(s)$ is an equilibrium is not too hard.
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Expected First Price Auction Revenue

- Revenue is highest bid $b(s^{1:n})$; expected revenue is $\mathbb{E}\left[b(S^{1:n})\right]$.

$$b(s) = s - \frac{\int_0^s F^{n-1}(\tilde{s})d\tilde{s}}{F^{n-1}(s)} = \frac{1}{F^{n-1}(s)} \int_0^s \tilde{s}F^{n-1}(\tilde{s})d\tilde{s} = \mathbb{E}\left[S^{1:n-1} | S^{1:n-1} \leq s\right]$$

That is, if a bidder has signal s, he sets his bid equal to the expectation of the highest of the other $n-1$ values, conditional on all those values being less than his own.
The expected revenue is:

$$\mathbb{E}\left[b(S^{1:n})\right] = \mathbb{E}\left[S^{1:n-1} | S^{1:n-1} \leq S^{1:n}\right] = \mathbb{E}\left[S^{2:n}\right]$$

First and second price auction yield the same expected revenue!

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Revenue equivalence theorem

Theorem (Myerson 1981)

Suppose n bidders have private values $s_i, \cdots, s_n$ identically and independently distributed with cdf $F(\cdot)$.
Then any equilibrium of any auction game in which

1. the bidder with the highest value wins the object,
2. a bidder with value 0 gets zero profits,

generates the same revenue in expectation.

Risk neutrality is necessary for revenue equivalence

- Risk-averse agents
  - for bidders:
    Dutch, first-price sealed-bid $\geq$ Vickrey, English
    Compared to a risk neutral bidder, a risk averse bidder will bid higher ("buy" insurance against the possibility of loosing)
    (Utility of winning with a lower bid $<$ utility consequence of loosing the object)
  - For auctioneer:
    Dutch, first-price sealed bid $\leq$ Vickrey, English
- Risk-seeking agents
  - The expected revenue in third-price is greater than the expected revenue in second-price (English)
  - Under constant risk-attitude: $(k+1)$-price is preferable to k- price

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Results for non-private value auctions

- Dutch strategically equivalent to first price sealed bid
- Vickrey not strategically equivalent to English
- All four protocols (Dutch, English, Vickery, first) allocate item efficiently

Theorem: Revenue non-equivalence

With more than 2 bidders, the expected revenues are not the same:
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A little auction

- We are playing a second price, sealed-bid auction
- You are bidding for a bag of coins with less than $10,000$ Rappen
- The winner(s) will play the second highest bid and receive the amount in the jar
- Website: https://scienceexperiment.online/classroom/r/eDhNWt

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- You are bidding for TIHS
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  Website: https://scienceexperiment.online/classroom/r/SVB894
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The winner's curse

- In a common value auction each bidder must recognize that he/she wins the object only when he/she has the highest signal
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multiple indistinguishable items for sale

Examples:

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- Pork belies
- Trans-Atlantic backbone bandwidth from NYC to Paris
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- Auction of multiple, distinguishable items
- Bidders have preferences over item combinations
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Task: The UK wanted (in 2000) to allocate "air space" for 3G mobile usage

Why an auction is a good choice:

- Utility to companies unknown to government; auction is method most likely allocating resource to those who can use them most valuably (rather than for example a "competition")
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The role for auction theorists:
Hal Varian leads the economics team designing the auctioning of ad-space.

- Multi-unit
- Multi-item
- Dynamic element (repeated games)
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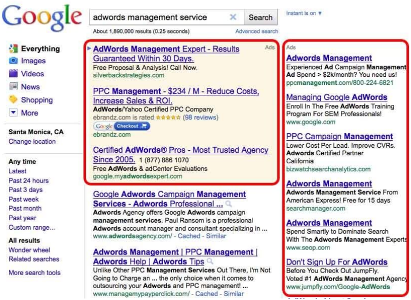

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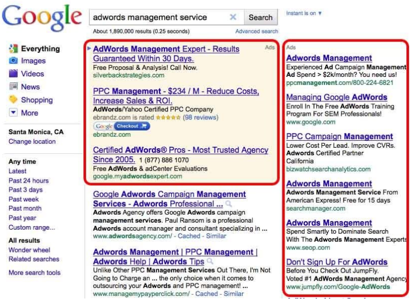

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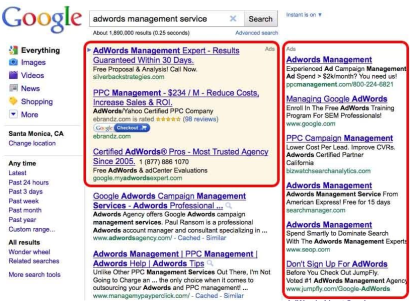

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Some introductory texts

- Vijay Krishna: Auction Theory (Academic Press)
- Paul Klemperer: Auction Theory: A guide to the literature (Journal of Economics Survey)
- Tuomas Sandholm COURSE: CS 15-892 Foundations of Electronic Marketplaces (CMU)