EVOLUTIONARY GAME THEORY

Heinrich H. Nax & Bary S. R. Pradelski

heinrich.nax@uzh.ch bpradelski@ethz.ch

University of Zurich[UZH]

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Centre National de la Recherche Scientifique

Common knowledge of rationality and the game

Suppose that players are rational decision makers and that mutual rationality is common knowledge, that is:

- I know that she knows that I will play rational
- She knows that “I know that she knows that I will play rational”
- I know that “She knows that “I know that she knows that I will play rational””
- ...

Further suppose that all players know the game and that again is common knowledge.

Rationality and the "as if" approach

- The rationalistic paradigm in economics (Savage, *The Foundations of Statistics*, 1954)
  - A person's behavior is based on maximizing some goal function (utility) under given constraints and information
- The "as if" approach (Friedman, *The methodology of positive economics*, 1953)
  - Do not theorize about the intentions of agents' actions but consider only the outcome (observables)
  - Similar to the natural sciences where a model is seen as an approximation of reality rather than a causal explanation (e.g., Newton's laws)

But is the claim right? Do people act (as if) they where rational?

Nash's mass-action interpretation (Nash, *PhD thesis*,1950)

"We shall now take up the ***"mass-action" interpretation of equilibrium*** *points. In this interpretation solutions have no great significance. It is* ***unnecessary to assume that the participants have full knowledge*** *of the total structure of the game, or the ability and inclination to go through* ***any complex reasoning*** *processes. But the participants are supposed to* ***accumulate empirical information on the relative advantages*** *of the various pure strategies at their disposal.*

...

Thus the assumption we made in this "mass-action" interpretation lead to the conclusion that the ***mixed strategies representing the average behavior*** *in each of the populations form an* ***equilibrium****."*

(bold text added for this presentation)

Nash's mass-action interpretation (Nash, *PhD thesis*,1950)

- A large population of identical individuals represents each player role in a game
- The game is played recurrently ($t = 0, 1, 2, 3, ...$):
  - In each period one individual from each player population is drawn randomly to play the game
- Individuals observe samples of earlier behaviors in their own population and avoid suboptimal play (successful strategies are copied more frequently)

Nash's claim: If all individuals avoid suboptimal pure strategies and the population distribution is stationary then it constitutes a [Nash] equilibrium

Almost true! Evolutionary game theory formalizes these questions and provides answers.

The folk theorem of evolutionary game theory

Folk theorem

- If the population process converges from an interior initial state, then for large t the distribution is a Nash equilibrium
- If a stationary population distribution is stable, then it coincides with a Nash equilibrium

Charles Darwin: "Survival of the fittest"
The population which is best adapted to environment (exogenous) will reproduce more

Evolutionary game theory
The population which performs best against other populations (endogenous) will survive/reproduce more

Domain of analysis

Symmetric two-player games

A symmetric two-player normal form game $G = \langle N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ consists of three object:

1. *Players:* $N = \{1, 2\}$, with typical player $i \in N$.
2. *Strategies:* $S_1 = S_2 = S$ with typical strategy $s \in S$.
3. *Payoffs:* A function $u_i : (h, k) \rightarrow \mathbb{R}$ mapping strategy profiles to a payoff for each player i such that for all $h, k \in S$:

$$u_2(h, k) = u_1(k, h)$$

Battle of the Sexes

	Cafe	*Pub*
Cafe	4, 3	0, 0
Pub	0, 0	3, 4

Not symmetric since:

$$u_1(Cafe, Cafe) \neq u_2(Cafe, Cafe)$$

Prisoner's dilemma

	Cooperate	*Defect*
Cooperate	$-1, -1$	$-8, \quad 0$
Defect	$0, -8$	$-5, -5$

Symmetric since:

$$\begin{aligned} u_1(\textit{Cooperate}, \textit{Cooperate}) &= u_2(\textit{Cooperate}, \textit{Cooperate}) &= -1 \\ u_1(\textit{Cooperate}, \textit{Defect}) &= u_2(\textit{Defect}, \textit{Cooperate}) &= -8 \\ u_1(\textit{Defect}, \textit{Cooperate}) &= u_2(\textit{Cooperate}, \textit{Defect}) &= 0 \\ u_1(\textit{Defect}, \textit{Defect}) &= u_2(\textit{Defect}, \textit{Defect}) &= -5 \end{aligned}$$

Symmetric Nash equilibrium

Definition: Symmetric Nash Equilibrium

A *symmetric Nash equilibrium* is a strategy profile σ^* such that for every player i,

$$u_i(\sigma^*, \sigma^*) \geq u_i(\sigma, \sigma^*) \text{ for all } \sigma$$

In words: If no player has an incentive to deviate from their part in a particular strategy profile, then it is Nash equilibrium.

Proposition

In a symmetric normal form game there always exists a symmetric Nash equilibrium.

Note: Not all Nash equilibria of a symmetric game need to be symmetric.

Evolutionarily stable strategy (Maynard Smith and Price, 1972)

Definition: Evolutionarily stable strategy (ESS)

A mixed strategy $\sigma \in \Delta(S)$ is an *evolutionarily stable strategy (ESS)* if for every strategy $\tau \neq \sigma$ there exists $\varepsilon(\tau) \in (0,1)$ such that for all $\varepsilon \in (0, \varepsilon(\tau))$:

$$U(\sigma, \varepsilon\tau + (1-\varepsilon)\sigma) > U(\tau, \varepsilon\tau + (1-\varepsilon)\sigma)$$

Let Δ^{ESS} be the set of evolutionarily stable strategies.

Alternative representation

Note that an ESS needs to be a best reply to itself, thus Δ^{ESS} is a subset of the set of Nash equilibria.

Proposition

A mixed strategy $\sigma \in \Delta(S)$ is an *evolutionarily stable strategy (ESS)* if:

$$\begin{aligned} U(\tau,\sigma) \quad &\leq \quad U(\sigma,\sigma) \;\; \forall\tau \\ U(\tau,\sigma) = U(\sigma,\sigma) \quad &\Rightarrow \quad U(\tau,\tau) < U(\sigma,\tau) \;\; \forall\tau \neq \sigma \end{aligned}$$

Prisoner's dilemma

	Cooperate	*Defect*
Cooperate	$-1, -1$	$-8, \quad 0$
Defect	$0, -8$	$-5, -5$

$$\Delta^{ESS} = \{\text{Defect}\}$$

Coordination game

	A	B
A	$4,4$	$0,0$
B	$0,0$	$1,1$

- Nash equilibria:
  $(A,A),(B,B),(0.2\cdot A+0.8\cdot B,0.2\cdot A+0.8\cdot B)$
- All Nash equilibra are symmetric.
- But the mixed Nash equilibrium is not ESS:
  - *A* performs better against it!
- Note that the mixed Nash equilibrium is trembling-hand perfect.

Existence of ESS not guaranteed

Example: Rock, paper, scissors

	R	P	S
R	$0,\ \ 0$	$-1,\ \ 1$	$1, -1$
P	$1, -1$	$0,\ \ 0$	$-1,\ \ 1$
S	$-1,\ \ 1$	$1, -1$	$0,\ \ 0$

- Unique Nash equilibrium and thus symmetric: $\sigma = (\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S)$
- All pure strategies are best replies and do as well against themselves as σ does against them $\Rightarrow$ Not an ESS!

Relations to normal form refinements

Propositions

- If $\sigma \in \Delta(S)$ is weakly dominated, then it is not evolutionarily stable.
- If $\sigma \in \Delta^{ESS}$, then (σ, σ) is a perfect equilibrium.
- If (σ, σ) is a strict Nash equilibrium, then σ is evolutionarily stable.

Summary

- Evolutionary game theory studies mutation processes (ESS)
- The stable states often coincide with solution concepts from the "rational" framework
- Evolutionary game theory does not explain **how** a population arrives at such a strategy
  $\Rightarrow$ Learning in games and behavioral game theory

The "best" textbook: Weibull, *Evolutionary game theory*, 1995

THANKS EVERYBODY

Keep checking the website for new materials as we progress:
http://gametheory.online/project_show/9